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Tree-Level Violation of the Equivalence Principle

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Massive particles of spin 0 and 1 violate the equivalence principle (EP) at the tree level. On the other hand, if these particles are massless, they agree with the EP, which leads us to conjecture that from a semiclassical viewpoint massless particles, no matter what their spin, obey the EP. General relativity predicts a deflection angle of $2.63''$ for a nonrelativistic spinless massive boson passing close to the Sun, while for a massive vectorial boson of spin 1 the corresponding deflection is $2.62''$.

KEY WORDS: equivalence principle; massive scalar boson; massive vectorial boson.

1. INTRODUCTION

Gravitation has a unique feature: the gravitational charge (or mass) of a body equals its inertial mass. In other words, all bodies accelerate equally in a gravitational field. It is therefore impossible to distinguish by observation of falling bodies between the uniform acceleration of a noninertial frame and a uniform gravitational field in an inertial frame. The equality of gravitational and inertial mass led Einstein to the unshakable conviction that any theory of gravitation must respect the equivalence principle (EP) which asserts that no physical experiment whatever can distinguish between the two possibilities mentioned previously.

In consequence of the EP, light rays follow null geodesics and all light rays are deflected in a gravitational potential by the same angle. Miraculously things do not change at all when one goes a step further and analyzes the same issue from a semiclassical viewpoint. If one computes, for instance, the cross-section at the tree level for the scattering of photons by the Sun's gravitational field, treated as an external field, it is found from this result that for a photon just grazing the Sun's surface the deflection is $1.75''$, which is exactly the same as that given by

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Einstein's theory (Accioly *et al.*, 1998, 1999; Boccaletti *et al.*, 1967). Thus, one may say that neither dispersive light propagation nor tree-level dispersive photon propagation can be produced by a gravitational field which obeys Einstein's field equations.

On the other hand, in a series of papers on the photon propagation around a massive body in quadratic theories of gravitation it was shown recently that, unlike Einstein's gravity, quadratic gravity produces dispersive photon propagation (Accioly *et al.*, 2000a,b, in press; Accioly and Blas, 2001). To be more specific, quadratic gravity produces energy-dependent photon scattering. An interesting consequence of this fact is that gravity's rainbows and higher-derivative gravity can coexist without conflict (Accioly and Blas, 2001). In this sense quadratic gravity is closer to quantum electrodynamics than any currently known gravitational theory. In fact, dispersive photon propagation is a trivial phenomenon in the context of QED. Based on the fact that the rainbow effect which is present in quadratic gravity is undetectable nowadays, it is possible to find a new constraint on the value of the contribution of the quadratic part (Accioly and Blas, 2001). This is a very important result given the scarcity of observational constraints on gravitational theories. In addition, it was also found that the gravitational deflection predicted by quadratic gravity is always smaller than that predicted by Einstein's theory (Accioly *et al.*, 2000a,b, in press; Accioly and Blas, 2001). It is worth mentioning that the R^2 sector of the theory of gravitation with higher derivatives does not contribute anything to the gravitational deflection (Accioly *et al.*, 1998, 1999).

The preceding considerations lead us to raise the interesting and important question: Is it possible to produce energy-dependent scattering of massive particles, at the tree level, in the very context of general relativity? Our aim here is precisely to show that the alluded energy-dependent scattering can be produced within the framework of Einstein's theory leading thus to a tree-level violation of the EP.

The scattering of spinless massive particles by a static gravitational field, treated as an external field, is analyzed in Section 2, while the scattering of massive spin 1 particles is discussed in Section 3. Both of them produce energy-dependent propagation. A summary of the main results is presented in Section 4.

We use natural units throughout. In our convention the signature is $(+---)$. The curvature tensor is defined by $R^{\alpha}_{\beta\gamma\delta} = -\partial_\delta \Gamma^{\alpha}_{\beta\gamma} + \cdots$, the Ricci tensor by $R_{\mu\nu} = R^{\alpha}_{\mu\nu\alpha}$, and the curvature scalar by $R = g^{\mu\nu} R_{\mu\nu}$, where $g_{\mu\nu}$ is the metric tensor.

2. SCATTERING OF SPINLESS MASSIVE BOSONS IN GENERAL RELATIVITY

We consider here the scattering of a spinless massive boson by a static gravitational field generated by a localized source such as the Sun, treated as an external field. As is well known, in general relativity the gravitational field is defined by the action

$$S = \int \sqrt{-g}\left[\frac{2R}{\kappa^2} - \mathcal{L}_M\right] d^4x,$$

where $\kappa^2 = 32\pi G$, with G being Newton's constant, is the Einstein's constant and $\mathcal{L}_M$ is the Lagrangian density for the usual matter. In the weak-field approximation, i.e., $g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$, with $\eta_{\mu\nu} = \text{diag}(+1, -1, -1, -1)$, and in the de Donder gauge, the field equations related to the action in hand turn out to be

$$\Box\gamma_{\mu\nu} = -\frac{\kappa}{2}T_{\mu\nu},$$

where $\gamma_{\mu\nu} \equiv h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h$ and $T_{\mu\nu}$ is the matter tensor which describes the physical system under consideration in special relativity, i.e., disregarding the gravitational field. Indices are raised (lowered) using $\eta^{\mu\nu}(\eta_{\mu\nu})$. The general solution of the equation above for a point particle of mass M located at $\mathbf{r} = \mathbf{0}$ is

$$h_{\mu\nu}(\mathbf{r}) = \frac{M\kappa}{16\pi r}(\eta_{\mu\nu} - 2\eta_{\mu 0}\eta_{\nu 0}).$$

The Lagrangian density for the boson–external-gravitational-field interaction, in turn, is given by

$$\mathcal{L}_{\text{int}} = -\frac{\kappa}{2}h^{\mu\nu}(\mathbf{r})\left[\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}\eta_{\mu\nu}(\partial_\alpha\phi\partial^\alpha\phi - m^2\phi^2)\right],$$

where ϕ is a scalar field describing particles of mass m and spin 0, from which we obtain the corresponding vertex function

$$V(p, p') = -\frac{\kappa}{2}h^{\mu\nu}(\mathbf{k})[p_\mu p'_\nu + p_\nu p'_\mu + \eta_{\mu\nu}(m^2 - p\cdot p')],$$

where

$$h_{\mu\nu}(\mathbf{k}) \equiv \int d^3\mathbf{r}e^{-i\mathbf{k}\cdot\mathbf{r}}h_{\mu\nu}(\mathbf{r}) = \frac{\kappa M}{4\mathbf{k}^2}\eta_{\mu\nu} - \frac{\kappa M}{2}\frac{\eta_{\mu 0}\eta_{\nu 0}}{\mathbf{k}^2} \tag{1}$$

is the momentum space gravitational field. Here $p(p')$ is the momentum of the incoming (outgoing) boson and $|\mathbf{p}| = |\mathbf{p}'|$.

Consequently, the cross-section for the scattering of spinless massive bosons by a static gravitational field can be written as

$$\frac{d\sigma}{d\Omega} = \frac{4M^2G^2}{(1-\cos\theta)^2}\left[\frac{1 - \frac{m^2}{2E^2}}{1 - \frac{m^2}{E^2}}\right]^2,$$

where E is the energy of the incident boson and θ is the scattering angle. For small angles, this expression reduces to

$$\frac{d\sigma}{d\Omega} = \frac{16M^2G^2}{\theta^2}\left[\frac{1-\frac{m^2}{2E^2}}{1-\frac{m^2}{E^2}}\right]^2. \tag{2}$$

However, for small angles,

$$\frac{d\sigma}{d\Omega} = \left|\frac{r}{\theta}\frac{dr}{d\theta}\right|. \tag{3}$$

Eqs. (2) and (3) then yield

$$\theta = \frac{4MG}{r}\left[1+\frac{m^2}{2E^2}\right]. \tag{4}$$

Some comments are in order here:

(i) For a massless spin-0 particle passing close to the Sun $\theta = 1.75''$, which is exactly the same result that is obtained with a massless spin-1 particle. Therefore, we may conjecture that massless particles, no matter what their spin, do not violate the EP at the tree level. Of course, from a classical viewpoint all massless particles travel the same null geodesic and, in addition, they are deflected in a gravitational potential by the same angle.

(ii) From a semiclassical point of view massive spinless bosons do violate the EP.

(iii) The scattering of spinless massive bosons in a static gravitational field is dispersive. To be more specific, such a scattering is energy dependent.

(iv) Rewriting (4) as

$$\theta = \frac{4MG}{r}\left[1+\frac{m^2}{2(m^2+\mathbf{p}^2)}\right],$$

and restricting our attention to the case that the motion of the spinless massive boson is nonrelativistic, i.e. $\mathbf{p}^2 \ll m^2$, we come to the conclusion that in this limit

$$\theta = \frac{3}{2}\left(\frac{4MG}{r}\right).$$

Thus, general relativity predicts a deflection angle of $2.63''$ for a nonrelativistic massive particle of spin 0 passing at the Sun's limb.

(v) For a massive scalar boson of spin 0 just grazing the Sun's surface $\theta > 1.75''$.

3. SCATTERING OF MASSIVE VECTORIAL BOSONS OF SPIN 1 IN GENERAL RELATIVITY

We turn our attention now to the scattering of massive spin-1 particles. The usual procedure in quantum gravity for obtaining the vertex function in the absence of fermions is to start with the action functional describing the matter field and to write the metric tensor $g_{\mu\nu}(x)$ as

$$g_{\mu\nu}(x) = \eta_{\mu\nu} + \kappa h_{\mu\nu}(x). \tag{5}$$

Consequently, the Feynman rule for the interaction of a massive spin-1 particle with a static gravitational field, treated as an external field, is obtained from the action for a gravitational minimally coupled massive spin-1 field

$$S = \int d^4x \sqrt{-g} \left[-\frac{F_{\mu\nu}F^{\mu\nu}}{4} + \frac{m^2 A_\mu A^\mu}{2} \right],$$

where A^ν is a vector field describing particles of mass m and spin 1 and $F_{\mu\nu} \equiv \partial_\nu A_\mu - \partial_\mu A_\nu$, expanding around flat space using (5). This leads to

$$S_{\text{int}} = \int d^4x \left[-\frac{\kappa}{8}(-4h^{\mu\alpha}\eta^{\nu\beta} + h\eta^{\mu\alpha}\eta^{\nu\beta})F_{\alpha\beta}F_{\mu\nu} \right.$$
$$\left. + \frac{m^2\kappa}{4}(-2h^{\mu\alpha} + h\eta^{\mu\alpha})A_\alpha A_\mu \right].$$

Accordingly, the vertex function for the vector-boson-field–static-external-gravitational-field interaction takes the form

$$V_{\mu\nu}(p, p') = \frac{\kappa}{2} h^{\lambda\rho}(\mathbf{k})[-\eta_{\mu\nu}\eta_{\lambda\rho} p \cdot p' + \eta_{\lambda\rho} p_\nu p'_\mu + 2(\eta_{\mu\nu} p_\lambda p'_\rho - \eta_{\nu\rho} p_\lambda p'_\mu$$
$$- \eta_{\mu\lambda} p_\nu p'_\rho + \eta_{\mu\lambda}\eta_{\nu\rho} p \cdot p') + m^2(-2\eta_{\mu\lambda}\eta_{\nu\rho} + \eta_{\mu\nu}\eta_{\lambda\rho})],$$

where $p(p')$ denotes the momentum of the incoming (outgoing) vectorial boson and $h^{\lambda\rho}(\mathbf{k})$ is given by (1).

The unpolarized cross-section for this process is

$$\frac{d\sigma}{d\Omega} = \frac{1}{(4\pi)^2} \frac{1}{3} \sum_{r=1}^{3} \sum_{r'=1}^{3} \mathcal{M}_{rr'}^2,$$

with $\mathcal{M}_{rr'} = \epsilon_r^\mu(\mathbf{p})\epsilon_{r'}^\nu(\mathbf{p'})V_{\mu\nu}$, where $\epsilon_r^\mu(\mathbf{p})$ and $\epsilon_{r'}^\mu(\mathbf{p'})$ are the polarization vectors for the initial and final vectorial bosons, respectively. Noting that (Greiner and Reinhardt, 1993; Mandl and Shaw, 1994)

$$\sum_{r=1}^{3} \epsilon_r^\mu(\mathbf{k})\epsilon_r^\nu(\mathbf{k}) = -\eta^{\mu\nu} + \frac{\kappa^\mu\kappa^\nu}{m^2},$$

we promptly obtain

$$\sum_{r}\sum_{r'}\mathcal{M}^2_{rr'} = V_{\mu\nu}V_{\alpha\beta}\left[\eta^{\mu\alpha}\eta^{\nu\beta} - \frac{\eta^{\mu\alpha}p'^{\nu}p'^{\beta} + \eta^{\nu\beta}p^{\mu}p^{\alpha}}{m^2} + \frac{p^{\mu}p^{\alpha}p'^{\nu}p'^{\beta}}{m^4}\right].$$

Using much algebra we arrive at the following expression for the cross-section in hand

$$\frac{d\sigma}{d\Omega} = \frac{4}{3}\left(\frac{GM}{\mathbf{k}^2}\right)^2 [2(p\cdot p')^2 - 8E^2 p\cdot p' + 12E^4$$
$$+ 5m^4 - 4m^2E^2 - 4m^2 p\cdot p'],$$

which, for small angles, reduces to

$$\frac{d\sigma}{d\Omega} = \frac{8}{3}\left(\frac{GM}{1-\frac{m^2}{E^2}}\right)^2 \frac{1}{\theta^4}\left[6\left(1-\frac{m^2}{E^2}\right) + 2\theta^2\left(-1+\frac{m^2}{E^2}\right) + \frac{3}{2}\left(\frac{m}{E}\right)^4\right].$$

Now, taking into account that for small angles $\frac{d\sigma}{d\Omega} = |\frac{r}{\theta}\frac{dr}{d\theta}|$, we come to the conclusion that

$$r = 4GM\left[\left(1+\frac{1}{2}\frac{m^2}{E^2}\right)\left(\frac{1}{\theta} + \frac{\theta\;\ln\theta}{3}\right)\right]. \tag{6}$$

Therefore, the scattering of vectorial bosons of spin 1 in the context of semiclassical general relativity is energy-dependent and thus violate the EP.

In the nonrelativistic limit (6) can be rewritten as

$$\theta = \frac{3}{2}\left(\frac{4GM}{r}\right)\left(1+\frac{\theta^2\;\ln\theta}{3}\right).$$

Hence, the gravitational deflection predicted by general relativity for a nonrelativistic massive vectorial boson of spin 1 passing close to the Sun is $2.62''$.

4. FINAL REMARKS

We have shown that massive particles of spin 0 and 1, unlike their massless counterparts, violate the EP at the tree level. In this vein we believe that if we perform a computation similar to that presented in Sections 2 and 3 for massive particles of spin 1/2, 2, etc., we shall come to the conclusion that they too violate the EP at the tree level. Therefore, the following statement is quite plausible:

Massive particles, no matter what their spin, violate the EP at the tree level, whereas their massless counterparts do not.

Last but not least we remark that in the metric formalism concerning general relativity, particles follow geodesics lines, while in a field-theoretical treatment, it is the particle–graviton interaction that replaces this geometrical fact.

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REFERENCES

Accioly, A., Azeredo, A., de Rey Neto, E., and Mukai, H. (1998). *Brazilian Journal of Physics* **28**, 496.

Accioly, A., Azeredo, A., and Mukai, H. (2002). Propagator, tree-level unitarity and effective nonrelativistic potential for higher-derivative gravity theories in *D* dimensions. *Journal of Mathematical Physics* **29**, 473.

Accioly, A., Azeredo, A., Mukai, H., and de Rey Neto, E. (2000a). *Progress of Theoretical Physics* **104**, 103.

Accioly, A. and Blas, H. (2001). *Physical Review D* **64**, 067701.

Accioly, A., Mukai, H., and Blas, H. (2000b). *Nuovo Cimento B* **115**, 1235.

Accioly, A., Ragusa, S., de Rey Neto, E., and Mukai, H. (1999). *Nuovo Cimento B* **114**, 595.

Boccaletti, D., de Sabbata, V., Gualdi, C., and Fortini, P. (1967). *Nuovo Cimento A* **48**, 58.

Greiner, W. and Reinhardt, J. (1993). *Field Quantization*, Springer, Berlin.

Mandl, F. and Shaw, G. (1994). *Quantum Field Theory*, Wiley, New York.